Colloid Thermodynamics and Isospectral Graphs

In a recent letter[1] entitled, "Can One 'Hear' the Thermodynamics of a (Rough) Colloid," Duplantier obtained excellent results giving a geometric analysis of the free energy of an electric double layer near a colloid surface, entirely similar to that of Kac[2] and Fisher[3], who related the low order moments of the eigenspectrum of a smooth drum to its shape, and raised the intriguing question, "Can one hear the shape of a drum?" In like manner Duplantier showed that the free energy of a (rough) colloid, when considered as a function of the Debye-Hückel screening length λ_{DH} can be expressed in terms of the eigenmodes and the eigenfunctions of the Laplace operator, defined over the colloid, viewed roughly as a crumpled membrane. His analysis is particularly relevant to large λ_{DH}. From this he concludes correctly, among other things, that "one can 'hear' the colloid free energy in the same way as its shape ... " This conclusion is checked with a explicit computation for a rectangular colloid. Although certainly not explicitly so stated, this paper leaves the reader with the impression that one could "hear" the shape of the colloid from the free energy, in the same sense that one could hear the shape of a drum from its characteristic function. That is to say, the eigenspectrum of a drum (or the free energy of a colloid) uniquely determines its shape. The purpose of this comment is to point out that there are limitations to this conclusion. What Kac and Fisher discovered was that it was straight forward to extract from the low order moments of the eigenspectrum the area, the length of the perimeter and the number of holes for a drum. In the rough colloid case, Duplantier has found additional wedge contributions.

Fisher replaced the continuous drum case of Kac with a discrete drum model consisting of a regular lattice array of atoms coupled by Hooke's law force along the lattice bond lines to their nearest neighbors. Baker[4] analyzed this discrete model and found by scanning a catalogue of linear graphs with a small number of points and bonds that there exist a great many pairs (and even triplets and quadruplets) of isospectral graphs. A sample pair is shown in Fig. 1. The method of demonstration of the isospectral property is to note that it is sufficient for the traces of the zeroth through the nth powers of the graph adjacency matrices to agree for them to be isospectral. Here n is the number of vertices of the graphs. In the case illustrated in Fig. 1, these traces are just,

$$15,\ 0,\ 36,\ 12,\ 144,\ 100,\ 684,\ 700,\ 3584,\ 4692,\ 19996,\ 30976,\ 116412,$$
$$203424,\ 698356,\ 1334092,\ \cdots .$$

His results provides strong presumptive evidence that one can not always hear the shape of a drum. The same type of replacement can be made for a rough colloid. In this case, Fisher's criterion that the drum graph be embeddable in a planar lattice, can be replaced by the criterion that the graph be embeddable in a crumple membrane, in the sense of Duplantier. Since a smooth colloid is a special case of a rough colloid, Baker's results, in addition, constitute strong presumptive evidence that the thermodynamics do not uniquely determine the geometry of the colloid.

This work was performed under the auspices of the U. S. DOE.

George A. Baker, Jr.

Theoretical Division, Los Alamos National Laboratory

University of California, Los Alamos, N. M. 87545, USA

PACS numbers: 05.70.-a, 02.90.+p

REFERENCES

1. B. Duplantier, Phys. Rev. Letts. **66**, 1555 (1991).
2. M. Kac, Amer. Math. Monthly **73S**, 1 (1966).
3. M. E. Fisher, J. Combinatorial Theory **1**, 105 (1966).
4. G. A. Baker, Jr., J. Math. Phys. **7**, 2238 (1966).

Figure Captions

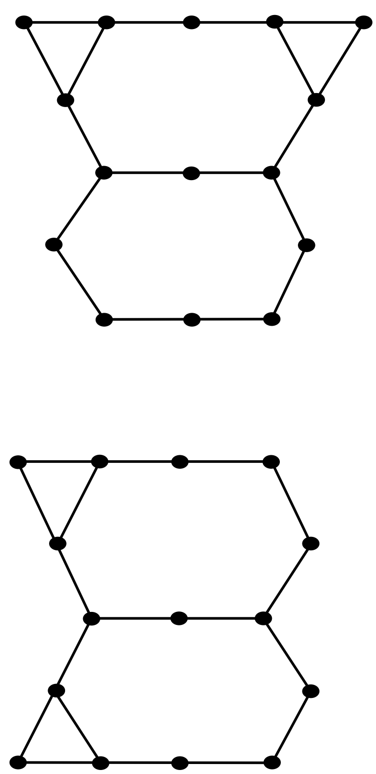

Fig. 1 A sample pair of isospectral drum graphs.